Multidisciplinary optimization formulation to the optimization of multirate systems

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Two approaches of multidisciplinary optimization are extended to a problem with a time-dependent model. The waveform relaxation method allows the modeling of a multirate system, then the multidisciplinary feasibility and the individual discipline feasibility strategies are used to define the optimization problem. With the second strategy, a way to obtain the jacobian of the operator associated to the waveform relaxation method is proposed.

***Index Terms*—Electromagnetic coupling, optimization methods, gradient methods.**

I. Introduction

Multidisciplinary optimization [1], [2] concerns the optimization of systems of coupled simulations. In this context, [3] gives three major approaches that can be applied: multidisciplinary feasibility (MDF), all-at-once (AA0) and individual discipline feasibility (IDF). In the MDF approach, the system is modeled by using a fixed-point strategy to couple the different submodels. At each optimization iteration, the model is evaluated several times until convergence of the fixed-point process. The AAO and IDF approaches avoid performing the fixed-point process at each evaluation of the model by adding some variables and constraints to the optimization problem.

For dynamic problems, waveform relaxation method (WRM) [4] allows to couple several models by applying a fixed-point strategy to waveforms. Furthermore, the time discretisations of the submodels can be different. Then, the WRM can generate the model used with the MDF approach. Nevertheless, if this model has a high computation time, it could be interesting to reduce the number of evaluations of the model, and to not do the fixed-point loop systematically by applying the IDF strategy. In this case, a discretised waveform is added as optimization variables, and the fixed-point criterion is added as optimization constraints. But the number of optimization variables is considerably extended. The large number of variables is problematic if a gradient-based method is used for the optimization, with a gradient computed by finite differences.

This article intends to apply the IDF approach to the optimization of a transformer modeled by WRM and to propose a way to obtain the gradient of the WRM application.

II. Waveform relaxation method

The WRM, also called *dynamic iteration*, solves iteratively r subsystems of differential algebraic equations to produce an approximation of the exact solution. At the k iteration and for the subsystem i, the following problem is solved:

$$\dot{\mathbf{y}}_i(t) = \mathbf{f}_i(\mathbf{Y}_i^k(t), \mathbf{Z}_i^k(t)) \tag{1}$$

$$0 = \mathbf{g}_i(\mathbf{Y}_i^k(t), \mathbf{Z}_i^k(t)), \tag{2}$$

with $\mathbf{Y}_i^k(t)$ and $\mathbf{Z}_i^k(t)$ depending on the solutions $\mathbf{y}^{k-1}(t), \mathbf{y}^k(t)$ of the differential equation (1) and the solutions $\mathbf{z}^{k-1}(t), \mathbf{z}^k(t)$ of the algebraic equation (2). Several schemes of relaxation exist to choose $\mathbf{Y}_i^k(t)$ and $\mathbf{Z}_i^k(t)$. With a Gauss-Seidel scheme, the subsystems are solved sequentially with

$$\mathbf{Y}_i^k(t) = [\mathbf{y}_1^k, \ldots, \mathbf{y}_{i-1}^k, \mathbf{y}_i^k, \mathbf{y}_{i+1}^{k-1}, \ldots, \mathbf{y}_r^{k-1}]^{\mathrm{T}}, \tag{3}$$

and

$$\mathbf{Z}_i^k(t) = [\mathbf{z}_1^k, \ldots, \mathbf{z}_{i-1}^k, \mathbf{z}_i^k, \mathbf{z}_{i+1}^{k-1}, \ldots, \mathbf{z}_r^{k-1}]^{\mathrm{T}}. \tag{4}$$

By introducing the fixed-point operator $\mathbf{\Psi}$ [5], the solution at the k iteration is written by

$$\begin{bmatrix}\mathbf{y}^k(t)\\ \mathbf{z}^k(t)\end{bmatrix} = \mathbf{\Psi}\left(\begin{bmatrix}\mathbf{y}^{k-1}(t)\\ \mathbf{z}^{k-1}(t)\end{bmatrix}\right). \tag{5}$$

Finally, the algorithm stops when $[\mathbf{y}^k(t), \mathbf{z}^k(t)]^{\mathrm{T}}$ is close enough to $[\mathbf{y}^{k-1}(t), \mathbf{z}^{k-1}(t)]^{\mathrm{T}}$. At the end of the iterative process, the solution is $[\mathbf{y}^K(t), \mathbf{z}^K(t)]^{\mathrm{T}} = \mathbf{\Psi}^K\left([\mathbf{y}^0(t), \mathbf{z}^0(t)]^{\mathrm{T}}\right)$.

In the IDF formulation, the $\mathbf{\Psi}$-operator will be approximated in $[\mathbf{y}(t), \mathbf{z}(t)]^{\mathrm{T}}$ by the first order Taylor expansion

$$\mathbf{\Psi}\left(\begin{bmatrix}\mathbf{y}_0(t)\\ \mathbf{z}_0(t)\end{bmatrix}\right) + \nabla\mathbf{\Psi}.\left(\begin{bmatrix}\mathbf{y}(t)-\mathbf{y}_0(t)\\ \mathbf{z}(t)-\mathbf{z}_0(t)\end{bmatrix}\right). \tag{6}$$

III. Multidisciplinary optimization

A. Multidisciplinary feasibility

Let us consider the optimization problem

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}} f(\mathbf{x}) \text{ such that } \mathbf{g}(\mathbf{x}) \leq 0. \tag{7}$$

We suppose that to obtain the output $\mathbf{g}(\mathbf{x})$, a system modeled by WRM is solved. With an MDF approach, at each evaluation of $\mathbf{g}$, the operator $\mathbf{\Psi}$ defined in (5) is applied K times on average. At the end of the optimization process, the number of evaluations of $\mathbf{\Psi}$ is approximately $n_{\mathrm{eval}} \times K$, with n_{eval} the number of model evaluations.

B. Individual discipline feasibility

With an IDF approach, the initial problem (7) is modified. The discretized waveforms $[\mathbf{y}(t), \mathbf{z}(t)]^{\mathrm{T}}$ are added to the optimization variables and the fixed-point condition is also added as a constraint of the problem. Only one evaluation of $\boldsymbol{\Psi}$ is done per model resolution, the consistency of the coupling being guaranteed by the optimization at the end of the process. The optimization problem to solve is

$$[\mathbf{x}^\star, \mathbf{y}^\star, \mathbf{z}^\star] = \arg\min_{\mathbf{x},\mathbf{y},\mathbf{z}} f(\mathbf{x}, \mathbf{y}, \mathbf{z}) \text{ such that} \tag{8}$$

$$\mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{z}) \leq 0, \tag{9}$$

$$[\mathbf{y}, \mathbf{z}]^{\mathrm{T}} - \boldsymbol{\Psi}\left([\mathbf{y}, \mathbf{z}]^{\mathrm{T}}\right) = 0. \tag{10}$$

At the end of the optimization process, the number of evaluations of $\boldsymbol{\Psi}$ is $\tilde{n}_{\mathrm{eval}}$, with $\tilde{n}_{\mathrm{eval}}$ the number of model evaluations. The optimization duration will be reduced if $\tilde{n}_{\mathrm{eval}} < n_{\mathrm{eval}} \times K$.

IV. Application Example

A LC filter supplying a transformer is considered [6]. This device is modeled by WRM (Fig. 1), with a 2D finite element model (FEM) for the simulation of the transformer. At each WRM iteration, a current $i^{k-1}(t)$ is imposed as a source into the circuit model. Its resolution gives the voltage $v^k(t)$ imposed to the transformer, then the resolution of the FEM gives the current $i^k(t)$ that will be the current source of the circuit model at the next iteration. At each iteration $i^k(t) = \Psi(i^{k-1}(t))$, and so the algorithm produces $i^K(t) = \Psi^K(i^0(t))$.

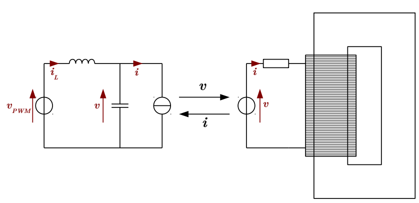

Fig. 1. Device split to apply the WRM.

The optimization aims to minimize the mass of the transformer by acting on the width L and the height H of the transformer. Moreover, the root mean square current i_{rms} into the transformer has to be equal to 3 A. The initial problem, solved with an MDF approach, is

$$\begin{cases} \min\limits_{H,L} m(H, L), \\ 20\text{cm} \leq H \leq 40\text{cm}, \; 12\text{cm} \leq L \leq 24\text{cm}, \\ H - \frac{2L}{3} > 0, \; i_{\mathrm{rms}} = 3 \; A. \end{cases} \tag{11}$$

We note $i^{\mathrm{out}}(t) = \Psi(i(t))$, $i_j = i(t_j)$ and $i_j^{\mathrm{out}} = i^{\mathrm{out}}(t_j)$ with t_j, $j = 1$ to n the time discretisation. With this notations, the problem to solve with the IDF formulation is

$$\begin{cases} \min\limits_{H,L,i} m(H, L), \\ 20\text{cm} \leq H \leq 40\text{cm}, \; 12\text{cm} \leq L \leq 24\text{cm}, \\ H - \frac{2L}{3} > 0, \; i_{\mathrm{rms}} = 3 \; A, \\ i_j - i_j^{\mathrm{out}} = 0, \forall j. \end{cases} \tag{12}$$

The discretized waveform of the current is added to the optimization variables and the constraints $i_j - i_j^{\mathrm{out}} = 0, \forall j$ ensure $i(t) - \Psi(i(t)) = 0$, namely the consistency of the coupling . One difficulty is to obtain the jacobian of the operator Ψ to use a gradient-based optimization algorithm. Due to the implicit Euler scheme used to solve the equations in time, the jacobian $\nabla\Psi$ is a triangular matrix. Indeed, $\frac{\partial i_q^{\mathrm{out}}}{\partial i_p} = 0$ if $q < p$. Moreover, we have $\frac{\partial i_p^{\mathrm{out}}}{\partial i_p} = \frac{\partial i_q^{\mathrm{out}}}{\partial i_q}$, $p \neq q$ and consequently $\frac{\partial i_{p+\ell}^{\mathrm{out}}}{\partial i_p} = \frac{\partial i_{q+\ell}^{\mathrm{out}}}{\partial i_q}$, $\forall \ell \geq 0$. The matrix $\nabla\Psi$ has identical values onto its diagonals. Finally, the computation of $\frac{\partial i_\ell^{\mathrm{out}}}{\partial i_1}$ is obtained for all ℓ by only one calculation by finite differences, and allows to have the complete matrix $\nabla\Psi$.

Optimizations using the sequential quadratic programming (SQP) method are done for the MDF and IDF formulations. SQP finds local optimum and needs the computation of the gradient of the objective and constraint functions. To compare the two approaches, 10 random initial points are generated. For 8 points, the IDF formulation converges to the same solution than the MDF one. But with the IDF approach, the number of evaluations of the Ψ operator is reduced (Fig. 2). So the number of resolutions of the FEM and the optimization duration are reduced too. The speedup factor is 3 on average, and even greater than 7 for one point.

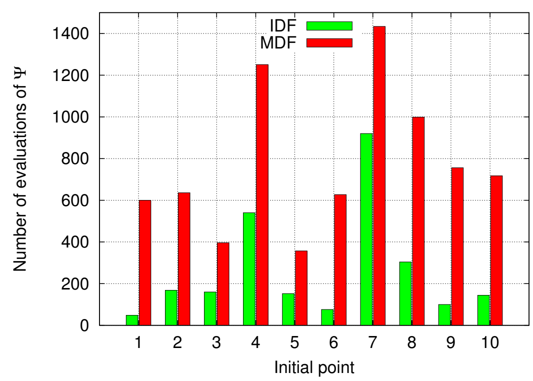

Fig. 2. Number of evaluations of Ψ operator

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